
MORE SIGNED NUMBERS AND EQUATIONS

□ **MORE ON OPPOSITES**

Sometimes we come across an expression like the following:

$$-(-4)$$

This can be read “the opposite of negative 4,” which we know is positive 4; that is, $-(-4) = 4$.

Advertisers take advantage of this idea of the “double negative” when they claim that “Nobody Doesn’t Like Sara Lee.”

$$-(-x) = x$$

This “double negative” idea can be used to solve a problem like this:

$$-(-7) + (-5) = 7 + (-5) = 2$$

What happens if we add a number and its opposite together? If we add 7 and its opposite, we get

$$7 + (-7) = 0 \quad (\text{start at 7; move 7 units to the left})$$

If we add -13 and its opposite, we get the same sum:

$$-13 + 13 = 0 \quad (\text{start at } -13; \text{ move 13 to the right})$$

It seems that

The sum of a number and
its opposite is zero.

We can write this as a formula. For any number n , whether it's positive, negative, or zero, we have:

$$n + (-n) = 0$$

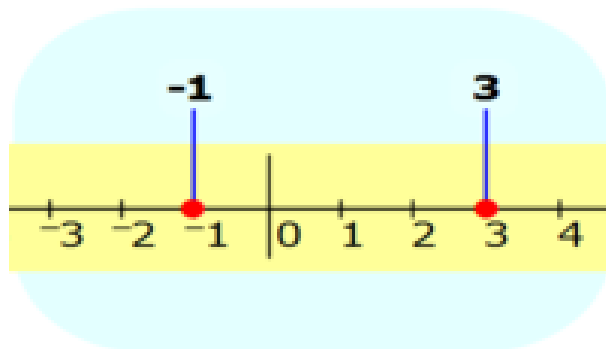
In fact, we can say that two numbers are opposites of each other precisely when their sum is 0. (This might explain to you why the opposite of 0 is 0.)

Homework

1. Simplify: $-(-9)$
2. Simplify: $-(-\pi)$
3. Prove that 23 and -23 are opposites of each other.
4. Prove that $-\pi$ and π are opposites of each other.
5. Prove that 0 and 0 are opposites of each other.
6. What is the opposite of -9 ? Explain.
7. What is the opposite of 123? Explain.
8. What is the opposite of 0? Explain.

□ **ANOTHER APPROACH TO ADDING AND SUBTRACTING**

We recall the method of using the number line to add and subtract positive and negative numbers: by starting at the first number, and then moving left or right, as appropriate, to arrive at the answer.



Another method, totally equivalent to the number-line method, is as follows:

If two adjacent signs are *opposites*, replace them with a single *minus* sign.

If two adjacent signs are the *same*, replace them with a single *plus* sign.

Then we proceed from there using the Jeopardy method, the number line, or common sense.

EXAMPLE 1:

A. $10 + (-3) = 10 - 3 = 7$

Change the two adjacent signs to a minus sign.

B. $-9 + (-5) = -9 - 5 = -14$

Change the two adjacent signs to a minus sign.

C. $8 - (-2) = 8 + 2 = 10$

Change the two adjacent signs to a plus sign.

D. $-7 - (-3) = -7 + 3 = -4$

Change the two adjacent signs to a plus sign.

E. $-10 - (-14) = -10 + 14 = 4$

Change the two adjacent signs to a plus sign.

EXAMPLE 2: Simplify: $-(-7) + 8 + (-10) - 3 - (-2)$

Here's the original expression:

$$-(-7) + 8 + (-10) - 3 - (-2)$$

Change the double negative in front to a positive 7:

$$7 + 8 + (-10) - 3 - (-2)$$

Change the adding of -10 to simple subtraction:

$$7 + 8 - 10 - 3 - (-2)$$

Change the subtracting of -2 to adding:

$$7 + 8 - 10 - 3 + 2$$

Now that all double signs have been removed, just proceed from left to right:

$$15 - 10 - 3 + 2$$

$$5 - 3 + 2$$

$$2 + 2$$

$$\boxed{4}$$

Homework

9. Evaluate (simplify) each expression:

a. $6 + (-6) + (-1)$

b. $-(-4) + 8 + (-8) + (-7)$

c. $-3 + (-4) + 5$

d. $-5 + (-5) + (-3)$

e. $12 + (-3) + (-12)$

f. $(-1) + (-2) + (-3)$

g. $-(-2) + (-2) + (-3) + 3$

h. $8 + 7 + 8 + (-1)$

10. Evaluate (simplify) each expression:

a. $-(-10) - 2$

b. $2 - 10$

c. $-3 - 8$

d. $-4 - 2$

- e. $5 - (-1)$ f. $10 - (-3)$ g. $-3 - (-3)$ h. $-1 - (-7)$
 i. $18 - 22$ j. $-7 - 5$ k. $-23 - (-4)$ l. $8 - (-2)$
 m. $-(-7) - 20$ n. $-99 - 1$ o. $-1 - (-2)$ p. $12 - (-4)$
 q. $13 - 21$ r. $-4 - 9$ s. $-(-12) - (-9)$ t. $-5 - (-1)$

11. Evaluate (simplify) each expression:

- a. $7 + (-3) - 8$ b. $-3 + 0 - (-1)$ c. $-8 - 2 - 1$
 d. $-2 - (-3) + 1$ e. $7 - (-3) - 8$ f. $9 - 2 - 3 - 4$
 g. $-2 + 5 + 9 - 1$ h. $-1 - (-1) - (-1)$ i. $8 + (-2) - 8 - (-1)$
 j. $-(-3) + 7$ k. $-(-1) - 3$ l. $-(-5) - (-4)$
 m. $-(-11) - (-8)$ n. $-(-7) - (+5)$ o. $-(-3) + (-3)$

12. True/False?

- a. The opposite of a negative number must be positive.
 b. The number 0 does not have an opposite.
 c. Every number has an opposite.

□ **MORE EQUATIONS**

Now we can solve a wider variety of equations. Let's start with the equation: $x + 9 = 7$, which has no positive solutions. We remove the 9 just as we would expect — subtract it from each side of the equation:

$$\begin{aligned} x + 9 &= 7 && \text{(the original equation)} \\ \Rightarrow x + 9 - 9 &= 7 - 9 && \text{(subtract 9 from each side)} \\ \Rightarrow x &= -2 && (9 - 9 = 0, \text{ and } 7 - 9 = -2) \end{aligned}$$

To check this solution, we see that $-2 + 9 = 7$ ✓

EXAMPLE 1: Solve each equation:

A. $n - 3 = -10$

Add 3 to each side of the equation:

$$n - 3 + 3 = -10 + 3 \Rightarrow n = -7$$

B. $y - 7 = -2$

Add 7 to each side of the equation:

$$y - 7 + 7 = -2 + 7 \Rightarrow y = 5$$

C. $z + 18 = 15$

Subtract 18 from each side of the equation:

$$z + 18 - 18 = 15 - 18 \Rightarrow z = -3$$

D. $a + 10 = -5$

Subtract 10 from each side of the equation:

$$a + 10 - 10 = -5 - 10 \Rightarrow a = -15$$

E. $5x - 30 = -20$

Add 30 to each side of the equation:

$$5x - 30 + 30 = -20 + 30$$

$$\Rightarrow 5x = 10$$

Last, divide each side of the equation by 5:

$$\Rightarrow x = 2$$

Homework

13. Solve each equation:

a. $x + 5 = 9$

b. $y - 13 = 12$

c. $n + 7 = 7$

- | | | |
|-------------------|---------------------|-------------------|
| d. $t - 8 = -4$ | e. $x - 5 = -3$ | f. $g - 9 = -13$ |
| g. $a + 8 = 7$ | h. $b + 9 = -2$ | i. $c + 7 = -10$ |
| j. $d - 9 = -10$ | k. $q - 19 = -10$ | l. $t - 9 = 9$ |
| m. $x - 17 = -2$ | n. $x + 17 = -3$ | o. $z + 7 = -4$ |
| p. $w - 11 = -11$ | q. $n - 2.3 = -0.7$ | r. $m + 0.3 = -2$ |

14. Solve each equation:

- | | | |
|------------------|-------------------|--------------------|
| a. $3a - 8 = -3$ | b. $4t - 3 = -1$ | c. $8w - 9 = -6$ |
| d. $2x - 4 = -2$ | e. $5n - 12 = -9$ | f. $6u - 10 = -5$ |
| g. $9w + 8 = 13$ | h. $6a - 7 = -4$ | i. $12g - 13 = -9$ |
| j. $7a - 9 = 14$ | k. $13w - 12 = 1$ | l. $12P - 12 = 14$ |

Review Problems

15. Evaluate (simplify) each expression:

- | | |
|-------------------------------------|---------------------------------|
| a. $-7 - (-3) - (-12)$ | b. $-(-8) - 10 + 3 + (-10)$ |
| c. $8 - (-2) - (-9)$ | d. $-5 + (-8) + 4 - 11$ |
| e. $5 + (-11) + 1 + (-8)$ | f. $-3 - (-2) - (-11) - 7 - 3$ |
| g. $-(-1) - 7 - 11 + (-5)$ | h. $-4 - (-7) + 12$ |
| i. $-(-2) - (-2) + (-7) - (-7) - 4$ | j. $11 + 6 + (-6) + 6$ |
| k. $-6 + 9 - 12 + 11$ | l. $5 - (-6) + (-4) - 7 - (-3)$ |
| m. $-2 + 2 + 9 - (-6) + 8$ | n. $-6 - 6 - 1$ |
| o. $6 - 4 - 7 - (-10)$ | p. $2 - (-12) + (-7)$ |

q. $-(-9) + 8 - (-5) + 9$

r. $6 + 2 - 8 + 4 + 7$

s. $-9 - (-5) + 10 + (-11)$

t. $-12 + 9 - 2 - (-5) - (-1)$

u. $-9 + 9 - 5 - 12$

v. $-2 - 11 - 3$

16. Solve each equation:

a. $x - 5 = 9$

b. $y + 13 = 12$

c. $n - 7 = 7$

d. $t + 8 = -4$

e. $x + 5 = -3$

f. $g - 9 = 13$

g. $a + 8 = -7$

h. $b + 9 = 2$

i. $c - 7 = -10$

j. $d + 9 = -10$

k. $q + 19 = -10$

l. $t - 9 = -9$

m. $x - 17 = 2$

n. $x - 17 = -3$

o. $z - 7 = -24$

17. Solve each equation:

a. $3x - 20 = -9$

b. $6n - 13 = 20$

c. $5q - 20 = -15$

d. $6a - 23 = -22$

e. $10w + 13 = 100$

f. $12p - 44 = -8$

g. $4T - 90 = -12$

h. $18a - 10 = -8$

i. $7x - 99 = -99$

j. $5g - 25 = -10$

Solutions

1. 9

2. π

3. $23 + (-23) = 0$ ✓

4. $-\pi + \pi = 0$ ✓ 5. $0 + 0 = 0$ ✓ 6. 9; because $-9 + 9 = 0$

7. -123 ; because $123 + (-123) = 0$

8. 0; because $0 + 0 = 0$

9. a. -1 b. -3 c. -2 d. -13
e. -3 f. -6 g. 0 h. 22

10. a. 8 b. -8 c. -11 d. -6 e. 6 f. 13
g. 0 h. 6 i. -4 j. -12 k. -19 l. 10
m. -13 n. -100 o. 1 p. 16 q. -8 r. -13
s. 21 t. -4

11. a. -4 b. -2 c. -11 d. 2 e. 2
f. 0 g. 11 h. 1 i. -1 j. 10
k. -2 l. 9 m. 19 n. 2 o. 0

12. a. T b. F, the opposite of 0 is 0 c. T

13. a. $x = 4$ b. $y = 25$ c. $n = 0$ d. $t = 4$ e. $x = 2$
f. $g = -4$ g. $a = -1$ h. $b = -11$ i. $c = -17$ j. $d = -1$
k. $q = 9$ l. $t = 18$ m. $x = 15$ n. $x = -20$ o. $z = -11$
p. $w = 0$ q. $n = 1.6$ r. $m = -2.3$

14. a. $a = \frac{5}{3}$ b. $t = \frac{1}{2}$ c. $w = \frac{3}{8}$ d. $x = 1$
e. $n = \frac{3}{5}$ f. $u = \frac{5}{6}$ g. $w = \frac{5}{9}$ h. $a = \frac{1}{2}$
i. $g = \frac{1}{3}$ j. $a = \frac{23}{7}$ k. $w = 1$ l. $P = \frac{13}{6}$

15. a. 8 b. -9 c. 19 d. -20 e. -13 f. 0 g. -22 h. 15
i. 0 j. 17 k. 2 l. 3 m. 23 n. -13 o. 5 p. 7
q. 31 r. 11 s. -5 t. 1 u. -17 v. -16

16. a. $x = 14$ b. $y = -1$ c. $n = 14$ d. $t = -12$ e. $x = -8$ f. $g = 22$

g. $a = -15$ h. $b = -7$ i. $c = -3$ j. $d = -19$ k. $q = -29$ l. $t = 0$
m. $x = 19$ n. $x = 14$ o. $z = -17$

17. a. $x = \frac{11}{3}$ b. $n = \frac{11}{2}$ c. $q = 1$ d. $a = \frac{1}{6}$ e. $w = \frac{87}{10}$ f. $p = 3$
g. $T = \frac{39}{2}$ h. $a = \frac{1}{9}$ i. $x = 0$ j. $g = 3$

“Education is not merely a means for earning a living or an instrument for the acquisition of wealth. It is an initiation into life of spirit, a training of the human soul in the pursuit of truth and the practice of virtue.”

– Vijaya Lakshmi Pandit